Mathematics 110: Lecture 4

Counting

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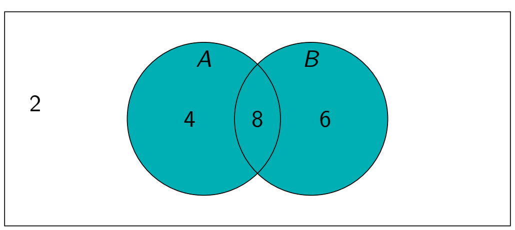
Example

- Suppose for a group of 20 students, 12 are taking a Spanish course, 14 are taking a biology course, and 8 are taking both a Spanish course and a biology course.
- How many students are taking either a biology or a Spanish course?
- Let U be the set of all 20 students, A be the set of students taking a Spanish course, and B be the set of students taking a biology course.
- Then $\mathrm{n}(U) = 20$, $\mathrm{n}(A) = 12$, $\mathrm{n}(B) = 14$, and $\mathrm{n}(A \cap B) = 8$.
- Filling in the Venn diagram, we see that
  - Since $\mathrm{n}(A \cap B) = 8$, $\mathrm{n}(A \cap B') = 4$.
  - Since $\mathrm{n}(A \cap B) = 8$, $\mathrm{n}(B \cap A') = 6$.
  - Hence $\mathrm{n}(A \cup B) = 4 + 6 + 8 = 18$.

Example (cont'd)

- The Venn diagram:



Example (cont'd)

- Note: Another way to see this result is to note that we must have

$$\mathrm{n}(A \cup B) = \mathrm{n}(A) + \mathrm{n}(B) - \mathrm{n}(A \cap B) = 12 + 14 - 8 = 18$$

Three counting formulas

- For any sets A and B,

$$n(A \cup B) = n(A) + n(B) - n(A \cap B).$$

- For any set A,

$$n(A') = n(U) - n(A).$$

- More generally, if $B \subset A$, then

$$n(A \cap B') = n(A) - n(B).$$

- From this it follows that, for any sets A and B,

$$n(A \cap B') = n(A) - n(A \cap B).$$

Example

- Suppose in a survey of 100 people,
  - 55 people said they like peppermint ice cream,
  - 40 people said they like anchovies on pizza, and
  - 20 people said they like peppermint ice cream and anchovies on pizza.
- Q: How many of the people surveyed do not like peppermint ice cream and do not like anchovies on their pizza?
- Let U be the set of all people surveyed, A be the set of people who like peppermint ice cream, and B be the set of people like anchovies on pizza.
- Then we want $n(A' \cap B')$.
- Since $n(A \cap B) = 20$, we see that

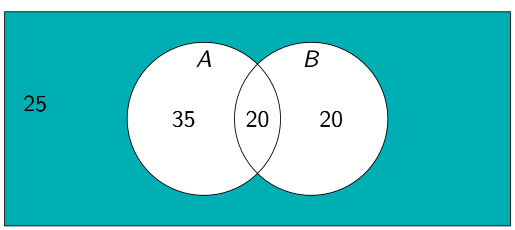
$$n(A \cup B) = 55 + 40 - 20 = 75.$$

- Thus

$$n(A' \cap B') = n((A \cup B)') = n(U) - n(A \cup B) = 100 - 75 = 25.$$

Example (cont'd)

- The Venn diagram:



Example

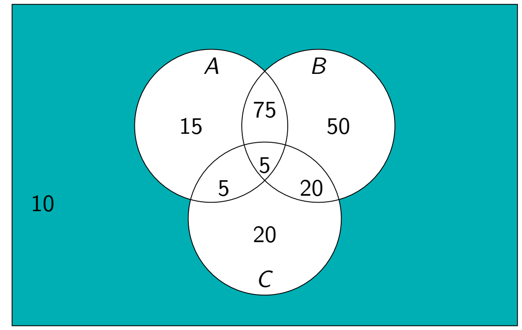
- Suppose 200 potential Democratic voters were asked to list which of the candidates, Sanders, Warren, and Harris, they would find acceptable as their nominee for President in 2020.
- Each voter could approve of zero, one, two, or all three of the candidates.
- Suppose
  - 100 listed Sanders,
  - 150 listed Warren,
  - 50 listed Harris,
  - 80 listed both Sanders and Warren,
  - 10 listed both Sanders and Harris,
  - 25 listed both Warren and Harris, and
  - 5 listed all three.
- Q: How many of the voters would not find any of the three candidates acceptable?

Example (cont'd)

- Let A be the set of voters who listed Sanders, B the set of voters who listed Warren, and C the set of voters who listed Harris.
- Then $n(A) = 100$, $n(B) = 150$, $n(C) = 50$, $n(A \cap B) = 80$, $n(A \cap C) = 10$, $n(B \cap C) = 25$, and $n(A \cap B \cap C) = 5$.
- Then
  - $n(A \cap B \cap C') = 80 - 5 = 75$,
  - $n(A \cap C \cap B') = 10 - 5 = 5$,
  - $n(B \cap C \cap A') = 25 - 5 = 20$.
- And so
  - $n(A \cap B' \cap C') = 100 - 85 = 15$,
  - $n(B \cap A' \cap C') = 150 - 100 = 50$,
  - $n(C \cap A' \cap B') = 50 - 30 = 20$.
- Hence $n(A \cup B \cup C) = 15 + 50 + 20 + 75 + 20 + 5 + 5 = 190$.
- And so $n((A \cup B \cup C)') = 200 - 190 = 10$, which is the number of voters who do not find any of the candidates acceptable.

Example (cont'd)

- The Venn diagram:



Example

- Suppose 200 potential Democratic voters were asked to list which of the candidates, Sanders, Warren, and Harris, they would find acceptable as their nominee for President in 2020.
- Each voter could approve of zero, one, two, or all three of the candidates.
- Suppose
  - 100 listed Sanders,
  - 150 listed Warren,
  - 50 listed Harris,
  - 80 listed Sanders and Warren,
  - 10 listed both Sanders and Harris,
  - 25 listed both Warren and Harris, and
  - 8 would not approve of any of the three.
- Q: How many of the voters would find all three candidates acceptable?

Example (cont'd)

- Let A be the set of voters who listed Sanders, B the set of voters who listed Warren, and C the set of voters who listed Harris.
- Then $n(A) = 100$, $n(B) = 150$, $n(C) = 50$, $n(A \cap B) = 80$, $n(A \cap C) = 10$, $n(B \cap C) = 25$, and $n((A \cup B \cup C)') = 8$.
- Let $x = n(A \cap B \cap C)$.
- Then
  - $n(A \cap B \cap C') = 80 - x$,
  - $n(A \cap C \cap B') = 10 - x$,
  - $n(B \cap C \cap A') = 25 - x$.
- And so
  - $n(A \cap B' \cap C') = 100 - ((80 - x) + (10 - x) + x) = 10 + x$,
  - $n(B \cap A' \cap C') = 150 - ((80 - x) + (25 - x) + x) = 45 + x$,
  - $n(C \cap A' \cap B') = 50 - ((25 - x) + (10 - x) + x) = 15 + x$.

Example (cont'd)

- Hence

$$n(A \cup B \cup C) = (10 + x) + (45 + x) + (15 + x) + (80 - x) + (10 - x) + (25 - x) + x$$
$$= 185 + x.$$

- And so $8 = n((A \cup B \cup C)') = 200 - (185 + x) = 15 - x$.
- Hence $x = 7$, the number of voters who would find all three candidates acceptable.

Example (cont'd)

- The Venn diagram: