Mathematics 140: Lecture 7

Tangent Lines

Dan Sloughter

Furman University

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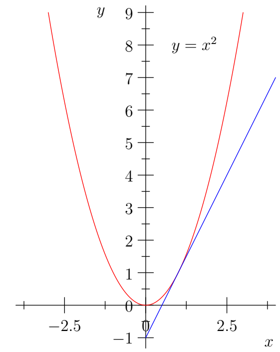
Example

- Problem: Find the equation of the line tangent to the graph of $f(x) = x^2$ at the point $(1, 1)$:



Example (cont'd)

- Note: If m is the slope of the desired tangent line, then the equation of the tangent line is

$$y = m(x - 1) + 1.$$

- Hence, to find the equation of the tangent line, we need find only its slope.
- Idea: Approximate the tangent line by *secant lines* which are close to the tangent line, that is, lines that intersect the graph of f at $(1, 1)$ and a point close to $(1, 1)$.
- A first approximation:
  - Consider the secant line through the points $(1, 1)$ and $(2, 4)$.
  - If this line has slope m_1, then

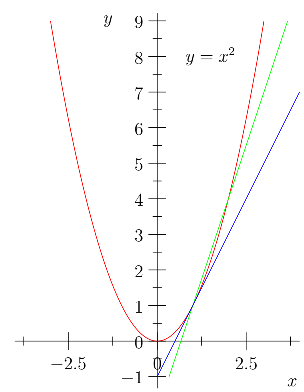
$$m_1 = \frac{4 - 1}{2 - 1} = 3.$$

  - Hence this secant line has equation

$$y = 3(x - 1) + 1.$$

Example (cont'd)

- Graphs of $f(x) = x^2$, the secant line $y = 3(x - 1) + 1$, and the tangent line:



Example (cont'd)

- Second approximation:
  - Now consider the secant line through $(1, 1)$ and $(1.1, (1.1)^2) = (1.1, 1.21)$.
  - If this line has slope $m_{0.1}$, then

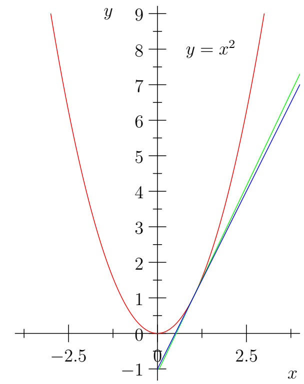
$$m_{0.1} = \frac{1.21 - 1}{1.1 - 1} = \frac{0.21}{0.1} = 2.1.$$

  - Hence this secant line has equation

$$y = 2.1(x - 1) + 1.$$

Example (cont'd)

- Graphs of $f(x) = x^2$, the secant line $y = 2.1(x - 1) + 1$, and the tangent line:



Example (cont'd)

- Third approximation:
  - Now consider the secant line through $(1, 1)$ and $(1.01, (1.01)^2) = (1.01, 1.0201)$.
  - If this line has slope $m_{0.01}$, then

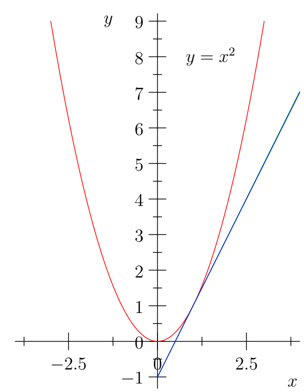
$$m_{0.01} = \frac{1.0201 - 1}{1.01 - 1} = \frac{0.0201}{0.01} = 2.01.$$

  - Hence this secant line has equation

$$y = 2.01(x - 1) + 1.$$

Example (cont'd)

- Graphs of $f(x) = x^2$, the secant line $y = 2.01(x - 1) + 1$, and the tangent line:



Tangent lines and secant lines

- Let C be the graph of $y = f(x)$.
- Let m be the slope of the line tangent to C at $(a, f(a))$.
- Idea: Let h be a small, nonzero, number, and approximate m by the slope of the secant line through the points $(a, f(a))$ and $(a + h, f(a + h))$.
- That is, we should have

$$m \approx \frac{f(a+h) - f(a)}{(a+h) - a} = \frac{f(a+h) - f(a)}{h}.$$

- Note:
  - We cannot evaluate this approximation when $h = 0$.
  - Thus we must find a way to determine m by consideration of the approximation for small, but nonzero, values of h.
  - However, the approximation is never equal to m, no matter how small we take h.