Mathematics 145: Lecture 10

Differentiable Functions

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September 21, 2016

Example

- Suppose $f(x) = \dfrac{1}{x}$.
- Then

$$\begin{aligned} f'(x) &= \lim_{h\to 0} \frac{f(x+h) - f(x)}{h} \\ &= \lim_{h\to 0} \frac{\frac{1}{x+h} - \frac{1}{x}}{h} \\ &= \lim_{h\to 0} \frac{\frac{x-(x+h)}{(x+h)(x)}}{h} \\ &= \lim_{h\to 0} \frac{-h}{x(x+h)h} \\ &= \lim_{h\to 0} \frac{-1}{x(x+h)} = -\frac{1}{x^2}. \end{aligned}$$

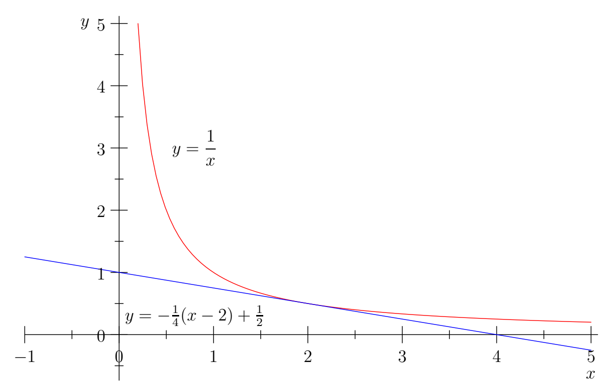
Example (cont'd)

- For example, $f'(2) = -\frac{1}{4}$.
- So the equation of the line tangent to the graph of $y = \dfrac{1}{x}$ at $x = 2$ is

$$y = -\frac{1}{4}(x-2) + \frac{1}{2}.$$

Example (cont'd)

- Graphs of $y = \dfrac{1}{x}$ with tangent line at $\left(2, \frac{1}{2}\right)$:



Continuity and differentiability

- Definition: We say a function f is *differentiable* on an open interval (a, b) if f is differentiable at every point in (a, b).
- Connection with continuity:
  - If f is differentiable at c, then

$$\lim_{h \to 0} \frac{f(c+h) - f(c)}{h} = f'(c).$$

  - Hence we must have $\lim_{h \to 0} f(c+h) = f(c)$.
  - Hence f must be continuous at c.

Example

- Let $f(x) = |x|$.
- To check if f is differentiable at 0, we evaluate, for $h \neq 0$,

$$\frac{f(0+h) - f(0)}{h} = \frac{|h|}{h} = \begin{cases} -1, & \text{if } h < 0, \\ 1, & \text{if } h > 0. \end{cases}$$

- Hence

$$\lim_{h \to 0^-} \frac{f(0+h) - f(0)}{h} = -1 \text{ and } \lim_{h \to 0^+} \frac{f(0+h) - f(0)}{h} = 1.$$

- So f is not differentiable at 0.
- Note: f is continuous at 0.