

Score: \_\_\_\_ out of 20 points

This assignment is due at the beginning of class on Friday, 3 February. The work you hand in must be your own. Show all your work!

- (8) 1. Compute the first four Picard iterates,  $y_0(t)$ ,  $y_1(t)$ ,  $y_2(t)$ , and  $y_3(t)$ , for the initial-value problem

$$\frac{dy}{dx} = 2x(1 + y), \quad y(0) = 1,$$

- (6) 2. Let  $y_1(x) = e^{2x} \cos(x)$  and  $y_2(x) = e^{2x} \sin(x)$ .  
(a) Compute the Wronskian  $W[y_1(x), y_2(x)]$ .

- (b) Are  $y_1$  and  $y_2$  linearly independent? Explain.

- (6) 3. Suppose  $a(x)$ ,  $b(x)$ , and  $c(x)$  are continuous on an interval  $I$ , with  $a(x) \neq 0$  for all  $x$  in  $I$ . Suppose  $y_1(x)$  and  $y_2(x)$  are both solutions of

$$a(x)y'' + b(x)y' + c(x)y = 0$$

on  $I$ . Moreover, suppose  $y_1(x_0) = 0$  and  $y_2(x_0) = 0$  for some  $x_0$  in  $I$ .

(a) Find  $W[y_1(x_0), y_2(x_0)]$ .

(b) Find  $W[y_1(x), y_2(x)]$  for any  $x$  in  $I$ .

(c) Are  $y_1$  and  $y_2$  linearly independent? Explain why or why not.